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NOTE ON DIRECTION.

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1. THE *path* of a moving thing is the sum total of all its positions.

2. The moving thing is called the *generatrix*.

3. The first and last positions of the generatrix are called *termini* of the path.

4. AXIOM. Whatever holds true as we approach indefinitely near a limit may be conceived of as true at the limit.

5. For brevity any *position* of the generatrix is called a generatrix.

6. An element of space is called a *point of space*; its limit, or mere position, a Geometrical point.

(*a*). The path of a point is called a *line*.
(*b*). " " " " line " " a *surface*.
(*c*). " " " " surface is " a *solid*.

7. The property of the motion of a generatrix which determines, and is is itself determined by, its next consecutive position, is called the direction of the path at that generatrix.

8. A *Straight path* is one which has constantly the same direction, i. e., one in which any other position of the generatrix can be taken instead of the next consecutive, in determining its direction; ∴ a St. P. is determined by any two of its genertarices. (Abbreviations, St. P., straight path, G., generatrix.)

9. A curved path is one that changes its direction at every generatrix.

10. (*a*), Any path determined by two of its generatrices is a St. P.

(*b*). The path of least extent is determined by its termini, ∴ it is a St. P. Special case. The shortest distance between two points is measured on a straight line.

11. Two St. Ps. can intersect in but one G., for if they did in two they would coincide throughout.

12. If one of two intersecting St. Ps. and their common G. be given, the other is determined, (*a*), by a G. in the St. P. to be determined; (*b*), by the *opening* between the paths. For we may suppose the two paths to have coincided at first, and then one of them to have been turned about their common G. (as a pivot or axis, as the case may be) until it contains the second G., and since the second G. determines the second path, it determines the opening, and conversely, the opening determines the path.

13. The opening between any path and a fixed path can be taken as the direction of that path.

14. Paths which have the same direction with respect to a *fixed* or initial path are called parallel paths.

15. Parallel paths can never intersect, for if they did they must form equal openings with a third path through the common G., and hence must coincide.

16. An opening can be measured by revolving a unit opening about an axis; ∴ it is proper to speak of the sum or difference of openings.

17. If the *initial* path pass through the intersection of two paths, the opening, called the *angle*, between them equals the difference of their directions. The two paths are called termini of the angle.

18. If two St. Ps. intersect, any two angles on opposite sides of one path and on the same side of the other are called adjacent angles, on opposite sides of both, opposite or vertical angles.

19. Equal adjacent angles are called *right angles*.

A right angle will be indicated by r.

20. If (1) and (2) are adjacent angles, then $(1) + (2) = 2\gamma$.

Proof. Revolve the common terminus of the ang's through an angle A,

$$\begin{aligned}&(1) \text{ becomes } (1) \pm A = (1)',\\ &(2) \quad \text{“} \quad (2) \mp A = (2)';\\ &\therefore (1) + (2) = (1)' + (2)' = \text{constant.}\end{aligned}$$

If $(1)' = (2)'$, by def. $(1)' + (2)' = 2r$; ∴ $(1) + (2) = 2r$; Q. E. D.

Cor. The sum of the angles about a G., on one side of a St. P., equals $2r$; on both sides, their sum equals $4r$.

21. The angle between two paths b and a, is indicated thus $\genfrac{}{}{0pt}{}{b}{a}$; the angle being positive when generated by motion from right to left.*

22. To show that this definition of an angle (17) is general, let a, b and c be three paths intersecting in three Gs.; then is

$$\genfrac{}{}{0pt}{}{a}{b} = \genfrac{}{}{0pt}{}{a}{c} - \genfrac{}{}{0pt}{}{b}{c}. \quad \text{Now } \genfrac{}{}{0pt}{}{b}{c} = -\genfrac{}{}{0pt}{}{c}{b}; \therefore \genfrac{}{}{0pt}{}{a}{b} = \genfrac{}{}{0pt}{}{a}{c} + \genfrac{}{}{0pt}{}{c}{b}.$$

Proof. Pass a St. P. β, through the intersection of a and c parallel to b.

By def. of parallel paths $\genfrac{}{}{0pt}{}{a}{\beta} = \genfrac{}{}{0pt}{}{a}{b}$ and $\genfrac{}{}{0pt}{}{c}{\beta} = \genfrac{}{}{0pt}{}{c}{b}$; but since a, β and c pass through a common G.,

$$\genfrac{}{}{0pt}{}{a}{\beta} = \genfrac{}{}{0pt}{}{a}{c} + \genfrac{}{}{0pt}{}{c}{\beta}; \therefore \genfrac{}{}{0pt}{}{a}{b} = \genfrac{}{}{0pt}{}{a}{c} + \genfrac{}{}{0pt}{}{c}{b} = \genfrac{}{}{0pt}{}{a}{c} - \genfrac{}{}{0pt}{}{b}{c}; \text{ Q. E. D.}$$

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*If a and b be the directions of the corresponding paths, by (17)

$$\genfrac{}{}{0pt}{}{b}{a} = b - a \text{ and } \genfrac{}{}{0pt}{}{a}{b} = a - b = -(b - a) = -\genfrac{}{}{0pt}{}{b}{a}.$$